

A new double-difference earthquake relocation algorithm for models with complex velocity structure

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Summary

Traditional double-difference approaches for earthquake relocation are established based on a least-squares framework that enables the minimization of the observed and modeled traveltime difference between earthquake pairs by iteratively adjusting the locations and origin times of a cluster of earthquake events. In these methods, construction of the inversion's model coefficient matrix, which governs the sensitivity of the traveltime to the model parameters (*i.e.*, earthquake location and origin time), requires deriving the partial derivatives of the traveltime of each earthquake with respect to its location and origin time. However, the underlying assumption that traveltime is always differentiable with respect to space is not always valid. One often needs to smooth or modify a velocity model in order to meet this assumption, but the modifications to a velocity model could alter the calculated traveltime and thus introduce bias into the inversion. To overcome this limitation, we develop a new double-difference earthquake relocation method that does not require the calculation of traveltime gradient and thus is applicable to models with complex velocity structure. Our results indicate that it is important to honor the true velocity structure in double-difference relocation to obtain reliable earthquake locations.

Method

The double-difference method of Waldhauser and Ellsworth (2000) has been widely used for earthquake relocation. This method is developed based on the assumption that the traveltime is always differentiable with respect to space so that one can formulate this problem as a least-squares inversion problem and solve it using a gradient-based iteration method. However, the presence of high velocity contrast (*e.g.*, a low/high velocity layer) in a velocity model can result in discontinuities in traveltime's spatial gradient, making a gradient-based inversion method numerically unstable. For example, Figure 1a shows a Howard County, TX, local velocity model, in which a low velocity zone appears at the depth of 2.7~2.8km and a high velocity Ellenberger layer is atop the basement layer at 3.5km depth (Fang et al., 2024). Figure 2a shows the first arrived P-wave travel time derived from full-wave modeling at 5 and 10km offsets when the event depth varies from 1 to 4km. The traveltime gradient with respect to depth shown in Figure 2b shows relatively smooth variations at 5km offset but exhibits drastic changes between the low and high velocity layers at

10km offset, indicating that the calculation of traveltime vertical gradient can be numerically challenging at these depths. The singularity in traveltime vertical gradient increases with increasing offset due to the presence of the high velocity Ellenberger layer. A common way to get around the issue caused by high velocity contrast is to remove the high contrast layers or smooth the velocity model to a certain extent so that traveltime is differentiable over the entire model. However, such modifications to the velocity model would introduce bias into the calculated traveltime and lead to incorrect relocation results (Micheline and Lomax, 2004). To overcome the limitation of traditional gradient-based double-difference method, we propose a new earthquake relocation method that is developed based on a stochastic inversion scheme.

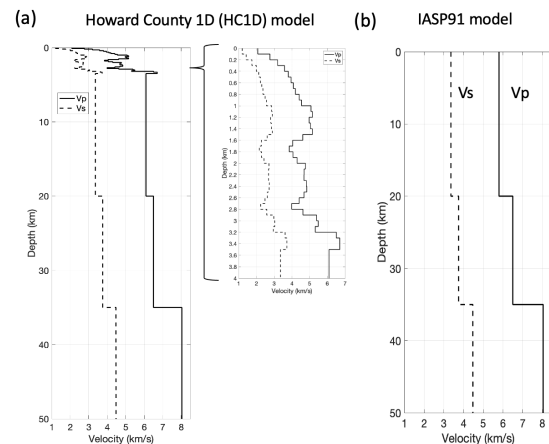


Figure 1: Howard County 1D model (HC1D) (a) and IASP91 model (b).

According to the definition of Waldhauser and Ellsworth (2000), the double-difference differential P and S-wave travel times between two events (the i -th and j -th events) can be written as

$$\Delta TP_k^{ij} = (TP_k^i - TP_k^j) - (tp_k^i - tp_k^j) \quad (1)$$

$$\Delta TS_k^{ij} = (TS_k^i - TS_k^j) - (ts_k^i - ts_k^j) \quad (2)$$

where TP_k^i and TS_k^i respectively represent the P and S-wave observed traveltimes of the i -th earthquake recorded at the k -th receiver, and tp_k^i and ts_k^i represent the corresponding modeled P and S-wave traveltimes. The least-squares cost function for minimizing the double-difference traveltimes can be written as

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$$C_{DD} = \sqrt{\frac{1}{2N_{pair}} \sum [(\Delta TP_k^{ij})^2 + (\Delta TS_k^{ij})^2]} \quad (3)$$

where $\sum[*]$ represents summation of N_{pair} event pairs (each pair is associated with two earthquakes recorded at one receiver). While we minimize the double-difference cost function of equation (3), we also need to retain the absolute traveltime relation from receivers to each earthquake event. The absolute traveltime constraints of all events used in double-difference inversion can be written as

$$C_T = \frac{1}{N_{event}} \sum \left\{ \sqrt{\frac{1}{2N_r} \sum_{k=1}^{N_r} [(TP_k^i - tp_k^i)^2 + (TS_k^i - ts_k^i)^2]} \right\} \quad (4)$$

where $\sum\{*\}$ in equation (4) represents summation over N_{event} events, and N_r is the number of receivers.

Note that the summation in equations (3) (common receiver) and (4) (common earthquake/event) applies to event-pair gather and event gather, respectively. By combining these two constraints together, we have the following total cost function:

$$f_{cost} = W_{DT} \cdot C_{DD} + W_{AT} \cdot C_T \quad (5)$$

W_{DT} and W_{AT} ($W_{DT} + W_{AT} = 1$) are the weights for double-difference traveltime constraint and absolute traveltime constraint, respectively. Because modeled traveltimes generally cannot perfectly match the traveltimes picked from the data due to velocity model inaccuracy and picking uncertainty, we must make some tradeoff between fitting of the double-difference traveltime and that of the absolute traveltime in the inversion. We can give more weight to C_T when we have a good local velocity model for the inversion, otherwise, we should weight C_{DD} more in order to eliminate the influence of velocity model inaccuracy.

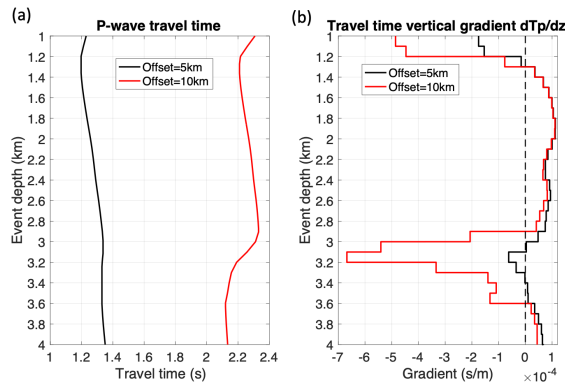


Figure 2: HC1D model's P-wave traveltime (a) and the corresponding vertical gradient (b) at 5 and 10km offsets.

Figure 3 illustrates our inversion workflow. We first use the differential evolution method (Storn and Price, 1997), which is a Monte Carlo random search approach, to search for an optimal solution (*i.e.*, relocated positions and origin times of all events) that minimize the cost function of equation (5) and then use grid search to search for a global offset for all

events that can minimize the absolute traveltime constraint of equation (4). We iterate and alternate over these two steps until we reach a predetermined maximum number of iterations or find a converged solution, which is defined as the solution whose maximum changes in events' locations from the previous iteration is less than a preset threshold value. The initial search ranges for each earthquake's location can be defined as the earthquake hypocenter's uncertainty ranges obtained from hypocenter inversion. In each subsequent iteration, we can progressively narrow down the ranges of the searched parameters based on the results of the previous iteration.

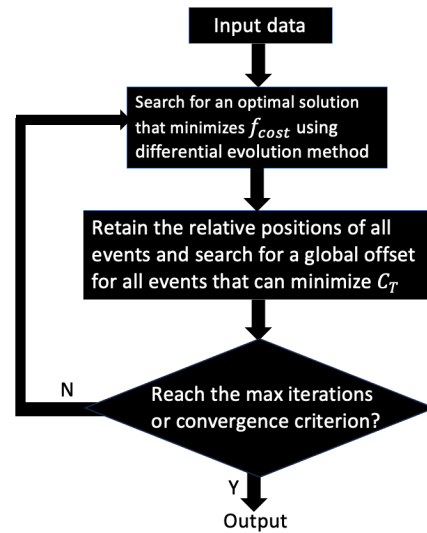


Figure 3: Stochastic double-difference inversion workflow. The input data are the picked first-break traveltimes of P and S arrivals.

Synthetic data example

We would like to understand the following two questions through numerical modeling:

- (1) When an inaccurate velocity model is used in hypocenter inversion, can the accuracy of events' relative positions be improved through double-difference relocation using the same velocity model?
- (2) Under the premise of the first question, can the results be improved using a more reliable velocity model for double-difference relocation?

We perform elastic full-wave modeling using the HC1D model (Figure 1a) for six earthquakes located at the same depth but at different horizontal positions, as illustrated in Figure 4. We picked the first-break P and S-wave arrival times from the full-wave modeling data and take these data as the observed data. We then use the IASP91 model (Figure

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1b) to invert for the earthquake locations using the workflow in Figure 3.

Comparison of the true earthquake locations and the inverted locations shown in Figure 5 indicates that using the incorrect IASP91 model in hypocenter inversion biases the epicenter locations by more than 1km and overestimate the earthquakes' depth by 3.8km. Accurate determination of the induced earthquake depths is critical to understand the earthquake triggering mechanism. Fang et al. (2024) pointed out that the use of an over-simplified velocity model in hypocenter inversion could often locate the event depth into the crystalline basement. Figure 6 shows comparisons of the double-difference relocated earthquake locations using the IASP91 and HC1D models. The earthquake locations relocated using the IASP91 model (blue squares) do not change much from the initial locations (black stars), indicating that the systematic bias in earthquake locations caused by the IASP91 model cannot be corrected through the double-difference relocation algorithm. For the results of using HC1D, we can see a significant improvement on the accuracy of the earthquake locations even without the inclusion of the absolute travel time constraint. We can recover the true earthquake locations when we increase the weight of the absolute traveltime constraint to 0.5 to suppress the error caused by the IASP91 model in the initial hypocenter inversion. From this numerical test, we have the following important observations and findings:

- (1) The influence of using an incorrect velocity model on earthquake locations cannot be removed by the double-difference relocation using the same velocity model (see Figure 6);
- (2) If the initial earthquake locations determined from hypocenter inversion are not reliable, an accurate velocity model is required for double-difference relocation (equation 5) to retrieve the true earthquake relative positions.

In summary, an accurate velocity model is important for both hypocenter inversion, which is used for inverting the initial location of individual earthquake, and double-difference relocation, which is applied to improve the relative positions of a cluster of earthquakes.

In terms of computational cost, the CPU time for the double-difference inversion (equation 5) of this synthetic example is about 4 minutes on an 8-Core Intel i7 workstation.

Field data example

We locate a cluster of earthquakes using the data recorded by the public seismic stations near Big Spring, Howard County, TX. We select 30 earthquakes (Figure 7) with high signal-to-noise ratio and good correlation with the adjacent earthquakes for double-difference relocation study. We use the HC1D model for both hypocenter inversion, which gives the initial earthquake locations, and double-difference

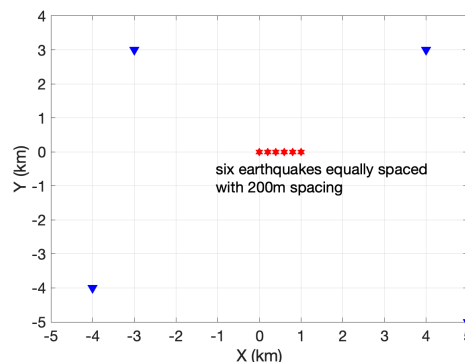


Figure 4: Model acquisition geometry. Six earthquakes (red stars) with 200m spacing spread along the x direction at the depth of 3km and four receivers (blue triangles) are located at different azimuths.

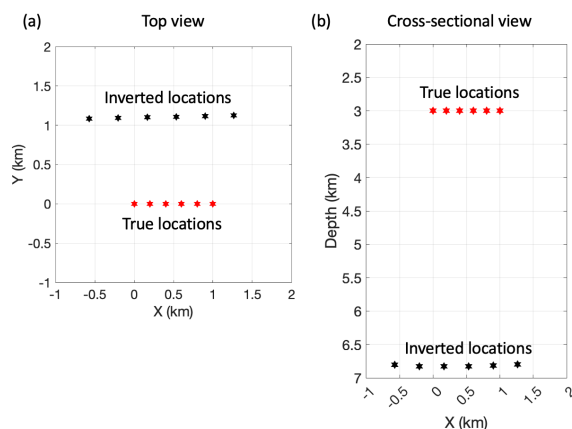


Figure 5: Earthquake locations inverted from hypocenter inversion using IASP91 model (black stars) vs. true earthquake locations (red stars).

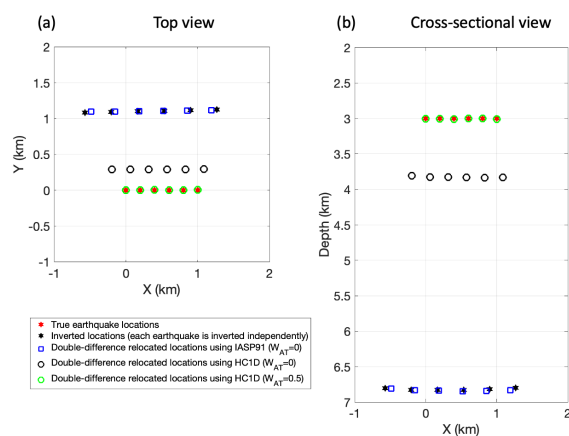


Figure 6: Comparisons of the double-difference relocated locations using the IASP91 (blue squares) and HC1D (black and green circles) models with different weights on the absolute traveltime constraint.

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relocation. The initial earthquake locations obtained from hypocenter inversion should have good accuracy since a local high-resolution velocity model is used. Thus, we set $W_{DT} = W_{AT} = 0.5$ in our double-difference inversion (equation 5) to have equal weights on the double-difference differential traveltimes constraint and the absolute traveltimes constraint. Figures 8 and 9 show the distribution of the initial earthquake locations inverted from hypocenter inversion (open circles) and the relocated locations after double-difference inversion (solid circles). Solid black lines indicate the relocation path of each earthquake. The earthquake locations tend to line up after double-difference relocation and give clear delineation of seismogenic zones. In double-difference inversion, we search 1 million random generations in each iteration. Each generation is randomly produced within the uncertainty ranges of each earthquake location. The fitting residual (equation 5) reaches 0.034s after 5 iterations, as shown in Figure 10. The iteration process converges rapidly, and the fitting residual is already below 0.04s after the first iteration. The total CPU time is 30 minutes on an 8-Core Intel i7 workstation. Such a computational time is not a significant challenge because double-difference relocation is only run occasionally when needed and there is no need to perform double-difference relocation in real-time.

Conclusions

We have developed a new double-difference earthquake relocation method based on a stochastic iteration framework. This method does not require the calculation of traveltimes gradient and is applicable to complex models with arbitrary velocity structure. We have shown both synthetic and field data examples to demonstrate the applicability of our newly proposed method. Through numerical modeling, we found that the error in earthquake locations caused by using an inaccurate velocity model in hypocenter inversion cannot be removed through double-difference relocation using the same velocity model. An accurate velocity model is necessary for both initial hypocenter determination and double-difference relocation.

Acknowledgements

We would like to thank Alan R. Huffman from HighPeak Energy Inc. for providing the Howard County velocity model used in this study.



Figure 7: Date and local magnitude of the selected earthquakes detected near Big Spring, TX, from October 2023 to March 2024.

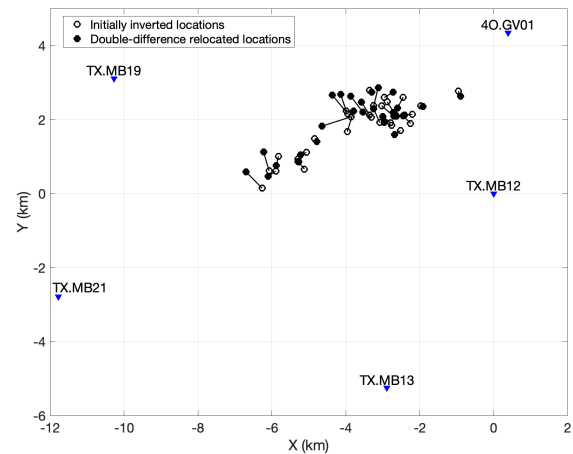


Figure 8: Mapview showing double-difference relocation of a cluster of 30 earthquakes detected using the public seismic stations near Big Spring, TX.

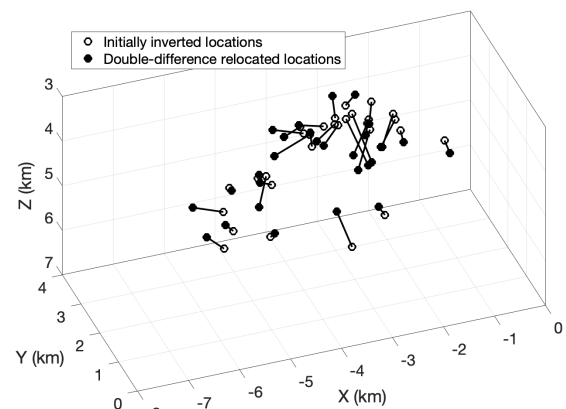


Figure 9: 3D view of the earthquakes shown in Figure 8.

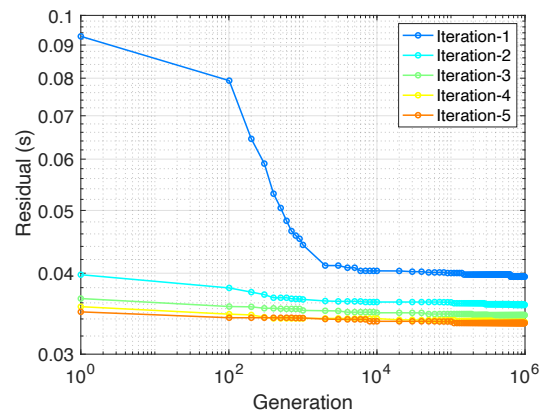


Figure 10: Convergence of the double-difference inversion iteration process.

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